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- XVII. The Effect of a Spherical Conducting Shell on the Induction at a Point in the Dielectric outside due to an Alternating Current in a Circular Circuit in the Dielectric inside, the Axis of the Conductor passing through the Centre of the Shell. By C. S. WHITEHEAD, M.A.*
- (1) It is well known that if there be an alternating current in a primary circuit sounds will be heard in a telephone in a secondary circuit, even if the latter be at a considerable distance from the former. If the sounds are preconcerted, signals may be, and in fact have been, transmitted in this manner on land through distances of four or five miles.

It has been suggested that the same plan could be employed to communicate with lightships. The primary is laid on the sea-bottom round the area over which the lightship swings, the ends of the cable being brought to land; the secondary is coiled round the lightship. I have been informed by Mr. S. Evershed that successful experiments were carried out by these means last summer in Dover harbour, the depth of the sea being about 1000 centims. To be able to judge whether a similar experiment will succeed in a greater depth of water, we must calculate the induction through the secondary circuit; sea-water being a conductor the investigation is somewhat complex, the final result, however, comes out in a simple form.

(2) The two following cases are considered:

CASE I.

A circular circuit carrying an alternating current is placed in the dielectric inside a spherical conducting shell, the axis of the circuit passing through the centre of the shell: to find the normal magnetic induction at any point in the dielectric outside the shell.

CASE II.

The circular circuit is placed on one side of an infinite conducting plate, the plane of the circuit being parallel to

* Read June 11, 1897.

the plate: to find the normal magnetic induction at any point on the other side of the plate.

In both cases the following result is arrived at:—

$$\frac{v_0}{u_0} = e^{-q\eta},$$

where v_0 is the maximum value of the normal magnetic induction at any point outside, u_0 the maximum value of the normal magnetic induction due to the current in the primary, supposing the conducting shell or plate absent, at the same point, η is the thickness of the shell or plate,

$$q = \left(\frac{2\pi\mu p}{\sigma}\right)^{\frac{1}{2}},$$

 μ is the permeability of the conducting shell or plate, σ its specific resistance, $p=2\pi$ times the frequency.

Let the frequency be 300, which makes p=1885; σ for sea-water = 2.10¹⁰ C.G.S. units, $\mu=1$.

If $\eta = 2000$ centims., which is about the depth of the sea by the North Sand Head Lightship off the Goodwin Sands, then

If
$$\frac{v_0}{u_0} = 21, \text{ and 79 per cent. is lost.}$$

$$\eta = 1000, \quad p = 1885,$$

$$\frac{v_0}{u_0} = 46, \text{ and 54 per cent. is lost.}$$

The above value of σ is from experiments on a sample of sea-water from the North Sea made by Mr. S. Evershed.

Take O the centre of the shell for origin, and axis of the circuit for axis of z.

Let D be the centre of the circuit, C any point in it.

OA = external radius of the shell = a.

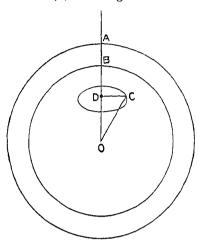
OB = internal radius of the shell = b.

$$OC = c, \qquad \angle DOC = \alpha.$$

 $a-b=\eta$.

Let P, Q, R be the components of the electromotive intensity parallel to the axes of x, y, z respectively, σ the specific resistance of the shell, μ the permeability of the shell, μ' that of the dielectric, K' the specific inductive capacity of

the dielectric, r, θ , ϕ the polar coordinates of any point, θ being the colatitude, ϕ the longitude.



Let the current in the circuit be represented by the real part of γe^{ipt} , $p = 2\pi \times$ frequency, $\iota = (-1)^{\frac{1}{2}}$.

By symmetry the induced currents in the shell will flow in circles parallel to the plane of the circuit.

Let Φ be the electromotive intensity tangential to one of these circles;

..
$$P = -\sin \phi \cdot \Phi$$
, $Q = \cos \phi \cdot \Phi$, $R = 0$ (1) Φ is by symmetry independent of ϕ .

Now P satisfies the equation

$$\nabla^2 P = \frac{4\pi\mu}{\sigma} \frac{dP}{dt}$$
 in the conductor,

and

$$\nabla^2 \mathbf{P} = \dot{\mu}' \mathbf{K} \frac{d^2 \mathbf{P}}{dt^2}$$
 in the dielectric.

Assume

٠.

P varies as
$$e^{ipt}$$
,

and let

$$\lambda^2 = -\frac{4\pi\mu\iota\rho}{\sigma},$$

$$\lambda'^2 = \mu' K' p^2.$$

$$\nabla^2 P + \lambda^2 P = 0 ; \qquad . \qquad . \qquad . \qquad . \qquad (2)$$

in the dielectric we must write \mathcal{N} for λ .

Q and R satisfy equations of the same form. Assume

$$P = \psi_{n}(\lambda r) \left(y \frac{d}{dz} - z \frac{d}{dy} \right) \chi_{n} e^{ipt}$$

$$Q = \psi_{n}(\lambda r) \left(z \frac{d}{dx} - x \frac{d}{dz} \right) \chi_{n} e^{ipt}$$

$$R = \psi_{n}(\lambda r) \left(x \frac{d}{dy} - y \frac{d}{dx} \right) \chi_{n} e^{ipt}$$

$$(3)$$

These equations make $\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz} = 0$, as should be the case.

 χ_n is an arbitrary solid spherical harmonic of degree n. Substituting in (2) we find

$$\frac{d^2\psi^n}{dr^2} + \frac{2(n+1)}{r} \frac{d\psi_n}{dr} + \lambda^2 \psi_n = 0. \quad . \quad . \quad (4)$$

Since R = 0, the last equation of (3) gives

$$\frac{d\mathbf{\chi_n}}{d\mathbf{\phi}} = 0.$$

 \therefore from the first of (3).

$$P = -\sin\phi \cdot \psi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{ipt}.$$

$$\therefore \Phi = \psi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{ipt}. \qquad (5)$$

Let $\omega =$ magnetic induction tangential to a meridian, v = magnetic induction along a radius.

Then from the theorem that the line integral of the electromotive intensity round a circuit is equal to the rate of decrease of the magnetic induction through the circuit, we obtain

$$\begin{split} \frac{d\omega}{dt} &= -\frac{1}{r} \frac{d(\Phi r)}{dr}, \\ \frac{dv}{dt} &= \frac{1}{r \sin \theta} \frac{d(\Phi \sin \theta)}{d\theta}. \end{split}$$

But Φ varies as e^{ipt} ,

$$\therefore \omega = -\frac{1}{\iota pr} \frac{d(\Phi r)}{dr},$$

$$v = \frac{1}{\iota pr} \frac{1}{\sin \theta} \frac{d(\Phi \sin \theta)}{d\theta}.$$

Since
$$\frac{d\chi_n}{d\phi} = 0, \quad \chi_n \text{ satisfies}$$

$$\frac{d}{dr} \left(r^2 \frac{d\chi_n}{dr} \right) + \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d\chi_n}{d\theta} \right) = 0,$$

$$\therefore \text{ from (5)}$$

$$\omega = -\frac{1}{\iota pr} \frac{d^2}{d\theta \cdot dr} \left\{ r \psi_n(\lambda r) \chi_n \right\} e^{\iota pt},$$

$$v = -\frac{1}{\iota pr} \psi_n(\lambda r) \frac{d}{dr} \left(r^2 \frac{d\chi_n}{dr} \right) e^{\iota pt}.$$
But
$$r \frac{d\chi_n}{dr} = n\chi_n,$$

$$\therefore \frac{d}{dr} \left(r^2 \frac{d\chi_n}{dr} \right) = n(n+1)\chi_n,$$
and
$$\frac{d}{dr} \left\{ r \psi_n(\lambda r) \chi_n \right\}$$

$$= \left\{ \frac{d \cdot r \psi_n(\lambda r)}{dr} + n \psi_n(\lambda r) \right\} \chi_n,$$

$$= \phi_n(\lambda r) \chi_n,$$
where
$$\phi_n(\lambda r) = \frac{d \cdot r \psi_n(\lambda r)}{dr} + n \psi_n(\lambda r), \qquad (6)$$

$$\therefore \omega = -\frac{1}{\iota pr} \phi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{\iota pt}$$

$$v = -\frac{n(n+1)}{\iota pr} \psi_n(\lambda r) \chi_n e^{\iota pt}$$

Again, μ' K' is very small (it is of the order 10^{-21}), hence, unless p is very large, we may neglect λ'^2 . Hence in the dielectric

$$\nabla^2 \mathbf{P} = 0,$$

we may therefore assume

$$\mathbf{P} = \left(y\frac{d}{dz} - z\frac{d}{dy}\right) \mathbf{X}^{n} e^{ipt},$$

where X_n is an arbitrary solid spherical harmonic, so that in the dielectric we must put $\psi_n(\lambda' r) = 1$, and, consequently, $\phi_n(\lambda' r) = n + 1$.

Therefore in the dielectric

$$\begin{aligned}
\mathbf{v} &= -\frac{n(n+1)}{\iota pr} \mathbf{X}_n e^{\iota pt} \\
\mathbf{\omega} &= -\frac{n+1}{\iota pr} \frac{d \mathbf{X}_n}{d\theta} e^{\iota pt}
\end{aligned} \right\} (8)$$

The inducing system is in the dielectric inside the shell. Hence in the dielectric inside the shell

$$v = -\frac{n(n+1)}{\iota pr} \left(X_n + X_{-n-1} \right) e^{\iota pt}$$

$$\omega = -\frac{1}{\iota pr} \left\{ (n+1) \frac{dX_n}{d\theta} - n \frac{dX_{-n-1}}{d\theta} \right\} e^{\iota pt}$$

In the dielectric outside the shell

$$\begin{aligned}
\nu &= -\frac{n(n+1)}{\iota pr} Z_{-n-1} e^{\iota pt} \\
\omega &= \frac{n}{\iota pr} \frac{d Z_{-n-1}}{d \theta} \epsilon^{\iota pt}
\end{aligned} \right\}, \qquad (10)$$

where Z_{-n-1} is an arbitrary solid spherical harmonic of degree -n-1.

In the conducting shell

$$v = -\frac{n(n+1)}{\iota pr} \psi_n(\lambda r) \chi_n e^{\iota pt}$$

$$\omega = -\frac{1}{\iota pr} \phi_n(\lambda r) \frac{d\chi_n}{d\theta} e^{\iota pt}$$
(11)

The boundary conditions are (1) that the normal magnetic induction, (2) that the tangential magnetic force is to be continuous. The tangential magnetic force $=\frac{\omega}{\mu}$ in the conductor, and $=\omega$ in the dielectric.

From these equations we obtain

$$\frac{X_n + X_{-n-1}}{Z_{-n-1}} = \frac{b^n \psi_n(\lambda b)}{a^n \psi_n(\lambda a)},$$

$$\frac{(n+1)X_n - nX_{-n-1}}{-nZ_{-n-1}} = \frac{b^n \phi_n(\lambda b)}{a^n \phi_n(\lambda a)};$$

$$\therefore b^n \{ (n+1)\phi_n(\lambda a)\psi_n(\lambda b) + n\phi_n(\lambda b)\psi_n(\lambda a) \} Z_{-n-1}$$

$$= a^n (2n+1)\phi_n(\lambda a)\psi_n(\lambda a) \psi_n(\lambda a) X_{-n-1}. \qquad (14)$$

Let λa and λb be so large that $\frac{1}{\lambda a}$, $\frac{1}{\lambda b}$ may be neglected $\psi_n(\lambda r)$ satisfies

$$\frac{d^2\psi_n}{dr^2} + \frac{2(n+1)}{r} \frac{d\psi_n}{dr} + \lambda^2 \psi_n = 0.$$

Assume

$$\psi_n = \frac{e^{mr}}{r^{n+1}}u,$$

and let

$$\lambda^2 + m^2 = 0,$$

$$\therefore \frac{d^2u}{dr^2} + 2m\frac{du}{dr} - \frac{n(n+1)}{r^2}u = 0.$$

Let

$$u = a_0 + \frac{a_1}{r} + \frac{a_2}{r^2} + \dots$$

Substituting, we find in the usual manner,

$$u=1-\frac{n(n+1)}{2mr}+\frac{(n-1)n(n+1)(n+2)}{1\cdot 2\cdot (2mr)^2}-\dots$$

where

$$m = + i\lambda$$

Hence, when $\frac{1}{\lambda r}$ is small,

$$\psi_n(\lambda r) = \frac{e^{\iota \lambda r}}{r^{n+1}} \text{ or } \frac{e^{-\iota \lambda r}}{r^{n+1}}.$$

Now Φ is zero at infinity and must decrease as r increases,

Substituting in (14)

$$Z_{-n-1} = \frac{(2n+1)be^{-i\lambda\eta}}{(n+1)a+nb} X_{-n-1}$$

$$= e^{-i\lambda\eta} X_{-n-1} (15)$$

if both a and b be large compared with their difference.

Let Ω denote the solid angle subtended by the circuit at any point;

V the potential due to the circuit;

U the normal magnetic force.

$$\Omega = 2\pi \sin^2 \alpha \sum_{n=1}^{\infty} \frac{1}{n+1} \left(\frac{c}{r} \right)^{n+1} P_n'(\alpha) P_n(\theta),$$

if r > c. P_n is the *n*th zonal harmonic.

$$V = \gamma e^{ipt} \Omega,$$

$$U = -\frac{dV}{dr},$$

$$= 2\pi \gamma \sin^2 \alpha e^{ipt} \frac{1}{r} \sum_{n=1}^{\infty} \left(\frac{c}{r}\right)^{n+1} P_n(\alpha) P_n(\theta). \qquad (16)$$

Now in the dielectric inside the shell,

But in the dielectric outside the shell

$$v = -\frac{n(n+1)}{\iota pr} Z_{-n-1} e^{\iota pt},$$

$$\therefore v = e^{-\iota \lambda^n} 2\pi \gamma \sin^2 \alpha e^{\iota pt} \frac{1}{r} \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta).$$

$$\lambda^2 = -\frac{4\pi \mu \iota p}{\sigma} = (1-\iota)^2 q^2,$$

Now

where

$$q^2 = \frac{2\pi\mu p}{\sigma},$$

.. real part of $e^{-i\lambda\eta}e^{ipt}$ = real part of $e^{-q\eta}e^{i(pt-q\eta)}$ = $e^{-q\eta}\cos(pt-q\eta)$,

$$\therefore v = e^{-q\eta} 2\pi \gamma \sin^2 \alpha \frac{1}{r} \left(\frac{c}{r}\right)^{n+1} P_n'(\alpha) P_n(\theta) \cos (pt - q\eta). \quad (19)$$

Let v_0 be the maximum value of v, U_0 that of U;

... from (16) and (19)

$$\frac{v_0}{\overline{\mathrm{U}}_0} = e^{-g\eta}. \quad . \quad . \quad . \quad . \quad (20)$$

The method employed in this investigation is taken from a paper by Professor H. Lamb, Phil. Trans. Part ii. 1883.

(4) Case II.

The result for this case may be deduced from the preceding by using a particular case of a transformation due to Professor C. Niven (Phil. Trans. Part ii. 1883).

He shows that if P_n denote a zonal harmonic of the *n*th degree, $s = \sin \theta$, $n = k\alpha$, $s = \frac{\rho}{\alpha}$; then when *n* and α become infinite, *k* and ρ remaining finite,

$$P_n = J_0(k\rho),$$

$$P'_n = \frac{ka^2}{\rho} J_1(k\rho),$$

Jo and J1 being Bessel functions.

To find the value of Ω in terms of Bessel's functions. Let P be any point, draw PM perpendicular to the axis of z.

Let CD, the radius of the circuit, =f,

 $PM = \rho$, $\angle POM = \theta$, $\angle COD = \alpha$, OC = c, DM = z, OP = r.

$$\Omega = 2\pi \sin^2 \alpha \sum_{1}^{\infty} \frac{1}{n+1} \left(\frac{c}{r} \right)^{n+1} P_n'(\alpha) \cdot P_n(\theta), \quad r > c.$$

Let $n = kr = k_1c$,

$$\sin \theta = \frac{\rho}{r}, \quad \sin \alpha = \frac{f}{c}.$$

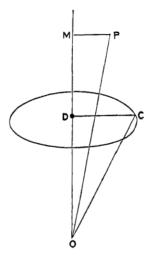
Let n, r, c become infinite, k, ρ, f remaining finite; \therefore ultimately $k=k_1$,

$$P_n(\theta) = J_0(k\rho), \quad P'_n(\alpha) = \frac{kc^2}{f}J_1(kf),$$

$$c = OD = OM - DM$$

$$= r - z \text{ ultimately };$$

$$\therefore \left(\frac{c}{r}\right)^n = \left(1 - \frac{z}{r}\right)^{kr} = e^{-kz} \text{ in the limit.}$$



The successive values of n are 1, 2, 3...; let k+dk be the successive values of k;

$$\therefore n+1 = (k+dk)r;$$

$$\therefore rdk = 1;$$

$$\therefore \frac{k}{n+1} {c \choose r} = \frac{n}{n+1} dk = dk;$$

$$\therefore \Omega = 2\pi f \int_0^\infty e^{-kz} J_0(k\rho) J_1(kf) dk.$$

This result can also be deduced from the equation

$$\frac{d^2\Omega}{dr^2} + \frac{1}{r}\frac{d\Omega}{dr} + \frac{d^2\Omega}{dz^2} = 0,$$

remembering that

when
$$z=0$$
, $\Omega=2\pi$ from $r=0$ to $r=a$.

$$\Omega = 0$$
 from $r = a$ to $r = \infty$.

We thus find

$$\begin{aligned} \mathbf{v} &= 2\pi \gamma e^{-q\eta} f \cos{(pt - q\eta)} \int_0^\infty k e^{-kz} \mathbf{J}_0(k\rho) \mathbf{J}_1(kf) dk, \\ \mathbf{U} &= 2\pi \gamma f \cos{pt} \int_0^\infty k e^{-kz} \mathbf{J}_0(k\rho) \mathbf{J}_1(kf) dk; \\ & \qquad \qquad \ddots \quad \frac{v_0}{\mathbf{II}} = e^{-q\eta} \end{aligned}$$

as before.

APPENDIX.

It may be useful to add the proof of the transformation used in Case II.

If $\mu = \cos \theta$,

Pn satisfies

$$(1-\mu^2)\frac{d^2P_n}{d\mu^2} - 2\mu\frac{dP_n}{d\mu} + n(n+1)P_n = 0. \quad . \quad (1)$$

Let

$$s = \sin \theta$$
.

(1) transforms into

$$(1-s^2)\frac{d^2P_n}{ds^2} + \frac{1-2s^2}{s}\frac{dP_n}{ds} + n(n+1)P_n = 0.$$

Assuming

$$P_n = a_0 + a_1 s + a_2 s^2 + \dots$$

we find in the usual manner

$$P_n = a_0 \left\{ 1 - \frac{n(n+1)}{2^2} s^2 + \frac{(n-2)n(n+1)(n+3)}{2^2 \cdot 4^2} s^4 - \dots \right\}.$$

But

$$P_n=1$$
 when $s=0$;

$$\therefore a_0 = 1.$$

Let

$$n = ka$$
, $s = \frac{\rho}{a}$,

and let n and a become infinite, k and ρ remaining finite.

$$\frac{n(n+1)}{2^2} s^2 = \frac{k^2 \rho^2}{2^2},$$

ultimately

$$\frac{(n-2)n(n+1)(n+3)}{2^2 \cdot 4^2} s^4 = \frac{k^4 \rho^4}{2^2 \cdot 4^2},$$
&c. = &c.
$$\therefore \text{ ultimately}$$

$$P_n = 1 - \frac{k^2 \rho^2}{2^2} + \frac{k^4 \rho^4}{2^2 \cdot 4^2} - \frac{k^6 \rho^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots$$

$$= J_0(k\rho).$$
Again
$$s^2 + \mu^2 = 1;$$

$$\therefore sds = -\mu d\mu,$$

$$\mu^2 = 1 - s^2 = 1 \text{ ultimately},$$

$$s = \frac{\rho}{a}; \quad \therefore ds = \frac{d\rho}{a};$$

$$\therefore d\mu = -s \frac{d\rho}{a} = -\frac{\rho d\rho}{a^2};$$

$$\therefore \Gamma_n^l = \frac{dP_n}{d\mu} = -\frac{a^2}{\rho} \frac{dJ_0(k\rho)}{d\rho}$$

$$= \frac{ka^2}{\rho} J_1(k\rho).$$

Discussion.

Mr. Evershed referred to some experiments of his own, from which he concluded that the author's formula gave too low an estimate of the attenuation; the discrepancy indicated that some term had been neglected.

Mr. YULE doubted whether the equations given by the author were quite applicable to sea-water. There was need, apparently, of a term involving the polarisation of the medium.

Mr. Heaviside communicated a criticism of the paper. It was not necessary to investigate the problem for any particular form of circuit from which the waves proceed. The attenuating factor for plane waves, due to Maxwell, was sufficient. Taking the best-known value for the conductivity of sea-water, there was no reason why the conductivity should interfere with signalling. A considerably greater conductivity must be proved for sea-water before it could be accepted that the failure of experiments on telegraphic communication with light-ships from the sea-bottom was due to that factor. It was unlikely theoretically, and Mr. Stevenson had contradicted it from a practical standpoint. For some reason, the account of the light-ship experiments had not been published, so that there was no means of finding the real cause of failure.

XVIII. On the Decomposition of Silver Salts by Pressure. By J. E. MYERS, M.Sc., Ph.D., late 1851 Exhibition Science Scholar, and F. Braun, Ph.D., Professor of Physics in the University of Strassburg i.E.*

As was demonstrated by Carey Lea† some years ago, silver salts and others may be decomposed by the application of pressure. By pounding a portion of the salt in a mortar one may readily effect decomposition.

A convincing proof is obtained by compressing a halogen salt of silver in the absence of sunlight and then subjecting the compressed mass to the developing and fixing processes of photography. A black residue of finely divided silver is the result. Exposure of the salt to sunlight, before compression, makes of course this test useless: the difficulty may be overcome by employing a mixture of AgNO₃ and KR (R=Cl, Br, or I) in the first instance.

The decomposition is most marked in the case of bromide of silver.

Two different metals immersed in liquid bromine constitute a galvanic element. The couple Ag, Br, Pt has an E.M.F. of about 0.95 volt. It appeared therefore of interest to

^{*} Read June 11, 1897.

[†] Phil. Mag. xxxi. p. 323 (1891); xxxiv. p. 46 (1892); xxxvi. p. 351. (1893); xxxvii. pp. 31 and 470 (1894).

[†] F. Braun, Wied. Ann. xvii. p. 610 (1862).